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We give a path integral derivation of the annulus diagram in a supersymmetric theory of open and closed strings with Dbranes. We compute the pair correlation function of Wilson loops in the generic weakly coupled supersymmetric flat spacetime background with Dbranes. We obtain a $-u^4/r^9$ potential between heavy nonrelativistic sources in a supersymmetric gauge theory at short distances.

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I. INTRODUCTION

Quantitative universal predictions for the low energy limit of String/M Theory that are independent of specific backgrounds or compactifications are hard to come by. In this paper, we take a step in the direction of quantitative universality, extracting a universal numerical result for the pair potential at short distances between heavy nonrelativistic sources in a supersymmetric gauge theory on some generic D-manifold [1,2] background of the type I or type II string theories. The result applies irrespective of whether the manifold, $\mathcal{D}$, is the worldvolume of a higher dimensional Dbrane, the intersection of multiple Dbranes, or the bulk transverse space orthogonal to some configuration of branes.

Our result is obtained from a path integral prescription for the pair correlation function of Wilson loops living in some D-manifold in a weakly coupled background of the type II theory, based on the earlier works [3–7]. We give a boundary reparameterization invariant computation of the supersymmetric pair correlation function of Wilson loops in the open and closed fermionic string theory [8–12] with Dbranes [13,2]. The normalization of the one-loop string vacuum amplitude in such a background can be determined from first principles following a classic method due to Polchinski [4]. However, it will be apparent from our result that the prediction of a short distance potential originating in fluctuations in the vacuum energy density is likely to hold in the broader context of the generic background of String/M theory. Our prescription for determining the phase ambiguities in the fermionic string path integral is derived from the imposition of infrared consistency conditions which follow from matching to an appropriate supergravity theory, the low energy theory at long distances. It is motivated in part by ideas taken from [14,15,1] and by the more unified description of self-duality and background fields that appears in the recent papers [16].

In an earlier paper with Chen [7] we obtained the short distance potential between heavy sources in a gauge theory in some generic background of the bosonic string. The potential is extracted from a covariant string path integral representation of the pair correlation function of Wilson loops. Our results apply both in the background with d=26 spacetime dimensions, or in the presence of a generic background for the Liouville theory with fewer matter fields, $c_m<d$. We find an attractive, and scale invariant, $1/r$ short distance interaction between the heavy gauge theory sources. We note that the bosonic string has a tachyon, which must either be stabilized (see the recent discussions in [17]), or eliminated— as is possible in the type I and type II string theories. The bosonic results are a useful warm-up for the computation of the annulus in stable backgrounds of the superstring. They also capture the correct qualitative features of the short distance potential in a background of the superstring with a tachyon instability. The calculation in the bosonic string proceeds as follows [7].

We consider heavy sources in the gauge theory in relative collinear motion with r^2=R^2+$v^2\tau^2$, v<<1, thus giving a simple realization of coplanar loops while mimicking straight-line trajectories in the Euclideanized X^0, X^1 plane. Here r is their relative position, and τ is the zero mode of the Euclideanized time coordinate, X^0. The scattering plane is wrapped into a spacetime cylinder by periodically identifying the coordinate X^0. Then the closed world-lines of the heavy sources are loops singly wound about this cylinder. We identify these closed world lines with Wilson loops. Following the earlier works of Alvarez [3], and of Cohen et al [5], we give a path integral prescription for the pair correlation function of Wilson loops. The loops can be taken to lie in the world-volume of a higher dimensional

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Dpbrane. Taking the large loop length limit of the correlation function, $L_i \simeq L_f \simeq T \rightarrow \infty$, with R held fixed, we define an effective potential as follows:

$$< M(C_i) M(C_f) > = -i \lim_{T\rightarrow\infty} \int_{-T}^{+T} d\tau V_{\rm eff.}[r(\tau), u] \quad . \tag{1}$$

The dominant contribution to the potential between the sources at short distances is from the massless modes in the open string spectrum. Suppressing the tachyon, and restricting to the massless modes of the bosonic string, the potential can be expressed as a systematic double expansion in small velocities and short distances with the result [7]:

$$V_{\rm bos.}(r, u) = -(d-2)\frac{1}{r} + O(z^2, uz/\pi, u^2) \quad , \tag{2}$$

where z is the dimensionless scaling variable, $z{=}r^2_{\rm min.}/r^2$, and $r^2_{\rm min.}{=}2\pi\alpha' u$ is the minimum distance scale probed in the collinear scattering of the heavy point sources. The $1/r$ static term receives velocity dependent corrections which are parameterized by the dimensionless variables, z^2, uz/π, and u^2. We show that the small velocity short distance approximation is self-consistent for distances in the regime, $2\pi\alpha' u << r^2 << 2\pi\alpha'$, and velocities in the range, $u << u_+$, where the upper bound, u_+, on permissible velocities can be estimated as described in [7]. Thus, String/M theory predicts velocity dependent corrections to the potential between two heavy sources in relative slow motion in a gauge theory, the numerical coefficients of which are given by a systematic expansion.

Evidence of a distance scale shorter than the string scale was originally found in the nonrelativistic scattering of D0branes [19,20], which gives a linear potential in the bosonic string theory [7]. The D0branes are assumed to have fixed spatial separation in a direction X^{d-1}, and in relative motion with nonrelativistic velocity v in an orthogonal direction X^d. The static linear potential between a pair of bosonic D0branes corresponds to a shift in the vacuum energy density relative to the background with no Dbrane sources due to a constant background electromagnetic vector potential [21,19] with vanishing electric field strength [16]: $A^\mu{=}\bar{A}^\mu$, $\mu{=}d{-}1$, $\partial_0 \bar{A}^\mu{=}0$. The systematics of the small velocity short distance double expansion, and the value $r^2_{\rm min.}{=}2\pi\alpha' u$ for the minimum distance probed in the scattering of D0branes, are in precise agreement with our results for the short distance potential between heavy gauge theory sources.

In this paper, we extend these results to the generic stable background of the supersymmetric type I and type II theories. We give a path integral prescription for the supersymmetric annulus amplitude, determining both its phase and its normalization from first principles. Boundary reparameterization invariance is imposed following the analysis of the bosonic string amplitude given in [7]. Our prediction for the short distance potential between heavy gauge theory sources in supersymmetric string theory is:

$$V_{\rm super.}(r, u) = -\frac{u^4}{r^9} 2^4 \pi^{7/2} \alpha'^4 \Gamma(\frac{9}{2}) + O(u^6) \quad , \tag{3}$$

which can be compared with the bosonic string result given in Eq. (2). The systematics of the velocity dependent corrections are much simpler in the fermionic string. This is a consequence of the BPS conditions or, equivalently, as in this example, of spacetime supersymmetry. The key ingredient which enables a prediction of the numerical coefficient in the short distance potential is its relationship to the vacuum energy computation in string theory: unlike in quantum field theories, the one-loop cosmological constant in critical string theory can be unambiguously normalized, an observation due to Polchinski [4].

We begin in section II with a brief discussion of supersymmetric boundary conditions and spin structure. Spinor conventions, and a recapitulation of the local symmetries of the world-sheet action for the fermionic string, are given in appendix A. Section III contains an evaluation of the supersymmetric annulus from first principles, beginning with the covariant path integral over world-sheets of specified spin structure and specified boundary condition on all of the world-sheet fields. The gauge fixing of the local world-sheet symmetries is carried out in section IIIA where we give a derivation of the supersymmetric annulus with boundaries on parallel and static Dbranes, up to undetermined phases. The precise global structure of the gauge orbit of the supersymmetry and super-Weyl transformations on the world-sheet is not known [6,2]. We will take the point of view that global ambiguities in the superstring path integral can be eliminated by the imposition of infrared consistency conditions that require matching to an appropriate supergravity theory describing the low energy physics. This prescription for the phases of the fermionic path integrals summed in the vacuum amplitude is given in section IIIB, using simple, and universal, infrared consistency conditions on the long distance physics. The result is an unambiguous determination of both the normalization and the phase of the supersymmetric annulus. Implementing boundary reparameterization invariance in the path integral as in [7], we derive an expression for the supersymmetric pair correlation function of Wilson loops in section IIIC. Finally, we extend our results in section IIID to generic boundary conditions corresponding to Dbranes in relative motion, imposing appropriate infrared consistency conditions as before. As a consistency check, we compute the short distance potential probed in the nonrelativistic small angle scattering of Dpbranes, recovering the numerical coefficient previously obtained in [2].

In section IV we adapt these results to the supersymmetric pair correlation function of Wilson loops corresponding to worldlines of heavy sources in relative slow motion. We show that the short distance potential between heavy sources in a supersymmetric gauge theory takes the form of a scale invariant $1/r$ fall-off contributed by the bosonic degrees of freedom, multiplicatively corrected by a convergent power series expansion in the dimensionless variable $z{=}r^2_{\rm min.}/r^2$. The leading term in the potential is given by the expression in Eq. (3). Some implications of our results are sketched in the conclusions.

II. SUPERSYMMETRIC BOUNDARY CONDITIONS AND SPIN STRUCTURE

We begin with the Brink-Di Vecchia-Howe-Deser-Zumino world-sheet action [8] used in Polyakov's path integral quantization of the fermionic string [10]:

$$S_{SP} = \frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{g}[\tfrac{1}{2}g^{mn}\partial_m X^\mu \partial_n X_\mu + \tfrac{1}{2}\alpha'\bar{\psi}^\mu\gamma^m\partial_m\psi_\mu + \sqrt{\tfrac{\alpha'}{2}}(\bar{\chi}_a\gamma^m\gamma^a\psi^\mu)(\partial_m X_\mu) + \tfrac{\alpha'}{4}(\bar{\chi}_a\gamma^b\gamma^a\psi^\mu)(\bar{\chi}_b\psi_\mu)] \quad , \tag{4}$$

invariant under both reparameterizations and local $N{=}1$ world-sheet supersymmetry transformations. The indices m,n=1,2 label the world-sheet coordinates for the string with metric g, and a,b=1, 2 label the flat local tangent space to the world-sheet. Spinor conventions and the local symmetries underlying the action are reviewed in the appendix. We use the label $\mu{=}0, \cdots, p$ for the Neumann directions parallel to the worldvolume of the Dpbranes. The branes are spatially separated by R in the X^9 direction, with $\mu{=}p{+}1, \cdots, 9$ labeling the Dirichlet directions orthogonal to the worldvolume of the Dpbranes. The boundary conditions on the embedding coordinates, X, and their fermionic superpartners, ψ, are obtained from the kinetic term in Eq. (4). The corresponding free field action in a flat embedding spacetime with component fermions, $\psi^{\pm}_{\mu}$, is [2]:

$$S[X,\psi] = \frac{1}{4\pi\alpha'}\int d^2\sigma[\partial^m X^\mu\partial_m X_\mu + \alpha'(\psi^{-\mu}(\partial_1 - i\partial_2)\psi^-_\mu + \psi^{+\mu}(\partial_1 + i\partial_2)\psi^+_\mu)] \quad , \tag{5}$$

which extends to the locally supersymmetric action given above. A variation of the classical action with respect to the embedding coordinate X gives the surface term:

$$\delta X^\mu(n^a\partial_a X_\mu) = 0 \quad . \tag{6}$$

As possible boundary conditions we list:

$$\begin{aligned} \mathrm{N} &: \quad n^a\partial_a X^\mu = 0 \\ \mathrm{D} &: \quad X^\mu = y^\mu \\ \mathrm{W} &: \quad \delta X^\mu \propto t^a\partial_a X^\mu \quad , \end{aligned} \tag{7}$$

where y^μ, $\mu{=}p{+}1, \cdots, 9$, gives the spacetime location of the Dbrane. The W, or modified Dirichlet (MD), boundary condition is motivated by the Wilson loop problem [3,7]. It permits fluctuations in the world-sheet fields tangential to the boundary. The boundaries of the world-sheet have been identified with the closed world-lines of a heavy quark–antiquark pair in the gauge theory.

A point source undergoing straight line motion with nonrelativistic velocity v in the X^0, X^1 plane with respect to the origin, $X^0{=}X^1{=}0$, and in zero external field, is described by the boundary conditions:

$$\mathrm{V} \quad : \quad n^a\partial_a(X^0 - vX^1) = 0, \qquad X^1 = vX^0 \quad , \tag{8}$$

with N (D) boundary conditions imposed on the X^0 (X^1) coordinates of the source fixed at the origin. The boundary conditions on two point sources in relative motion in a $\mathcal{D}$-manifold are rather simple, irrespective of whether the motion occurs within the worldvolume of a higher dimensional Dbrane, the intersection of two or more Dbranes, or in the bulk transverse space orthogonal to some configuration of Dbranes. The point sources are the end-points of open strings. Then we distinguish the d=10 embedding coordinates of the world-sheet as NN, ND, or DD, directions, depending on whether both, one, or neither, point source has nonvanishing spacetime momentum in the direction of the coordinate [13,2]. In the discussion that follows, we restrict ourselves to NN and DD coordinates alone. Note that identical boundary conditions must be imposed on all of the NN, and all of the DD, fermions in order to preserve the global $SO(1,p)\times SO(9-p)$ symmetry of the vacuum amplitude. The world-sheet gravitino satisfies the same boundary conditions as the NN fermions— this is dictated by supersymmetry.

Consider the variation of the world-sheet action with respect to the fermion field. The vanishing of the surface term dictates the following condition on fermion bilinears on the boundary:

$$\psi^{+\mu}(\delta\psi_{+\mu}) = \psi^{-\mu}(\delta\psi_{-\mu}) \quad , \tag{9}$$

with solutions, $\psi^{+\mu}{=}\pm\psi^{-\mu}$, at any boundary. As a check, we perform a supersymmetry transformation on the surface term. This gives the condition, $n_a[(\partial_b X_\mu)(\bar{\xi}\gamma^a\gamma^b\psi^\mu)]{=}0$. Align the world-sheet with coordinate σ^2 normal, and σ^1 tangential, to the boundary. Assume Neumann boundary conditions on the embedding coordinate X_μ: $\partial_2 X_\mu{=}0$, with $\partial_1 X_\mu{\neq}0$. Then the requirement that the boundary conditions on the ψ^μ preserve world-sheet supersymmetry implies the restriction, $\xi^+{=}\mp\xi^-$, on permissible supersymmetry transformations on the boundary. With the Dirichlet boundary condition, $\partial_1 X_\mu{=}0$, we obtain the constraint, $\bar{\xi}\psi^\mu{=}0$. Thus, choosing one or other sign for the NN fermions simultaneously determines the choice of phase for the DD fermions. In component form, the choice $\xi^+{=}\mp\xi^-$ implies that $\chi^{-\mu}{=}\mp\chi^{+\mu}$. The constraints on the fermionic fields for the W and V boundary conditions on the world-sheet are a straightforward generalization of this reasoning.

The annulus amplitude in a flat spacetime background of the supersymmetric string can be represented as a path integral summing over fluctuations of world-sheets with cylindrical topology weighted by the action in Eq. (4):

$$\mathcal{A} = \tfrac{1}{2}\sum_{\beta,\alpha\in 0,1} C^\beta_\alpha \int_{[\beta,\alpha]} \frac{[dX][d\psi][dg][d\chi]}{\mathrm{Vol}(gauge)} e^{-S_{SP}[X,\psi,g,\chi]-\mu_0\int_{\mathcal{M}} d^2\sigma\sqrt{g}-S_{\mathrm{ren.}}} \quad . \tag{10}$$

Thus, the world-sheets of the fermionic string are endowed with additional degrees of freedom, and the quantum fluctuations about some minimum action configuration summed in the path integral must include a consideration of these modes. We consider the simplest background configuration of static parallel Dpbranes separated by a distance R. The stretched string between the Dbranes contributes a term, $-R^2 l/4\pi\alpha'$, to the action in Eq. (10), further corrections being suppressed at weak coupling. Since the one-loop vacuum amplitude is a sum over surfaces of cylindrical topology with Euler characteristic $\chi{=}0$, the amplitude is free of any dependence on the string coupling constant. Also, we can drop boundary cosmological constant terms in favor of the bulk cosmological constant, μ_0, since these are not independent Lagrange parameters on the cylinder. We will gauge all of the local symmetries reviewed in appendix A. $S_{\mathrm{ren.}}$ contains any additional counterterms that may be necessary in order to obtain amplitudes invariant under both world-sheet diffeomorphisms and Weyl transformations of the metric, as well as local supersymmetry transformations and super-Weyl rescalings of the world-sheet gravitino. Divergent contributions to the path integral arising from local gauge anomalies in the measure will be absorbed in a renormalization of the bare couplings introduced in $S_{\mathrm{ren.}}$, including the bulk cosmological constant, the renormalized values being set to zero at the end of the calculation [4,5].

We turn next to global aspects of the path integral. We are summing over spin structures and averaging over the $\pm$ ambiguity in the boundary condition on the fermions at the boundary of the world-sheet. The label α on the path integral refers to a choice of spin structure on the world-sheet: the change in phase in a Weyl fermion upon traversal of a closed path homotopic to either boundary of the cylinder. Under $\sigma^1{\to}\sigma^1+1$, the left and right-moving component fermions transform as:

$$\psi^{\pm\mu}(\sigma^1+1,\sigma^2) = -e^{-\pi i\alpha}\psi^{\pm\mu}(\sigma^1,\sigma^2) \quad . \tag{11}$$

The parameter $\alpha{=}0$ (1) labels the string path integral computed with world-sheet spinors that are, respectively, anti-periodic (periodic) around the single closed cycle of the cylinder. The parameter β denotes the ambiguity, $\psi^{+\mu}{=}\pm\psi^{-\mu}$, at any boundary, described above. Specifically, for either NN or DD fermions, we will define β as follows:

$$\psi^{+\mu}(\sigma^1,0) = -e^{-\pi i\beta}\psi^{-\mu}(\sigma^1,0), \qquad \text{with } \psi^{+\mu}(\sigma^1,1) = \psi^{-\mu}(\sigma^1,1) \quad . \tag{12}$$

Choosing the phase at the $\sigma^2{=}1$ end-point to correspond to periodic fermions is pure convention. Thus, $\beta{=}0$ (1) corresponds to reflection with (without) a phase change of π in the fermionic wavefunction at the $\sigma^2{=}0$ end-point. We remark that this convention corresponds to that in the text [2]. In the critical dimension, and with an unambiguous prescription for the phases, C^β_α, of the path integrals, there will be no global gravitational or Lorentz anomalies. Our prescription for determining the absence of global phase ambiguities in the one-loop vacuum amplitude will be physical, rather than constructive. It is motivated by infrared consistency conditions on the long distance physics, as will be clarified in section IIIB. For the discussion in section IIIA, we encourage the reader to think of the C^β_α as *unspecified*, and therefore ambiguous, phases that weight the different contributions to the annulus amplitude.

III. PATH INTEGRAL EVALUATION OF SUPERSYMMETRIC ANNULUS

We will now give a path integral evaluation of the supersymmetric annulus— a sum over orientable world-sheets with boundaries on parallel Dpbranes, paying special attention to the imposition of world-sheet supersymmetry both in the bulk, and on the boundary. We begin with the simplest configuration of parallel and static Dpbranes, generalizing our results in section IIID for the V boundary conditions describing Dpbranes in relative motion. The gauge fixing of the local symmetries on the world-sheet, and a derivation from first principles of the annulus amplitude up to unknown phases, is described in section IIIA. In section IIIB, we show how infrared consistency conditions can be used to determine all of the phase ambiguities in the path integral. We extend this derivation in IIIC to an analysis of the supersymmetric pair correlation function of Wilson loops following the treatment in [7]. The extension to generic boundary conditions describing branes in relative motion is given in section IIID. This result will be adapted in section IV to give an expression for the supersymmetric pair correlation function of Wilson loops corresponding to the worldlines of heavy gauge theory sources in slow relative motion. The normalization and the phase of the pair correlation function is therefore precisely, and unambiguously, determined, allowing a derivation of the short distance potential between the sources.

A. Gauge Fixing of the Local World-sheet Symmetries

We begin by gauge fixing the Lorentz transformations in the local tangent space at any point in the world-sheet, thereby eliminating one of the four bosonic gauge parameters. This implies that, although it is convenient to write classically covariant expressions in terms of zweibeins e_m^a, the number of physical degrees of freedom in the path integral is the same as with metrics: we use local Lorentz rotations to eliminate one of the independent degrees of freedom in the zweibein. Likewise, we eliminate two of four independent modes of the gravitino by choosing super-conformal gauge, $\gamma^m \chi_m$=0 [9], thereby gauge fixing super-Weyl transformations. This immediately creates an apparent problem with supersymmetry since we have only two fermionic but three bosonic gauge parameters, but this is not so. Recall that we work in the critical spacetime dimension gauging Weyl transformations of the metric. As in the path integral quantization of the bosonic string [4], although it is convenient to keep the Weyl mode in the discussion of the measure, the principle of ultralocality requires that any explicit dependence on the Weyl mode only contribute local, renormalizable, terms to the effective action [4,5]. The unique choice for such terms is the Liouville action. Thus, in the critical dimension, the Weyl mode entirely decouples.

We will employ similar reasoning in gauging local Lorentz and super-Weyl transformations: ultralocality of the measure in the path integral [4] requires that any explicit dependence on the Weyl and super-Weyl modes can only contribute terms proportional to the supersymmetric Liouville action [10]. Thus, although we find it convenient to keep all four bosonic and fermionic gauge parameters in the discussion below, there are really half as many physical gauge parameters in the critical superstring corresponding, respectively, to diffeomorphisms and local supersymmetry transformations. Following gauge fixing, the path integrals reduce to ordinary integrals over constant modes and moduli. The counting of superconformal Killing spinors and supermoduli on a Riemann surface is given by the supersymmetric analog of the Riemann-Roch theorem. For cylindrical topology, the answer is rather simple, since both the superconformal Killing spinors and the supermoduli are simply constant spinors. These can only exist on a cylinder with both periodic spin structure and the periodic boundary condition at the endpoints of the open string.

The zweibein and metric are related by g_{mn}=$e_m^a e_n^b \delta_{ab}$. We make the same choice of fiducial world-sheet metric as in the analysis of the bosonic string:

$$ds^2 = l^2(d\sigma^1)^2 + (d\sigma^2)^2, \qquad 0 \le \sigma^1 \le 1, \quad 0 \le \sigma^2 \le 1 \quad . \tag{13}$$

Thus, the fiducial einbein on the boundary is $\hat{e}$=$\sqrt{g}$, and the modulus l can be identified as the boundary length, l=$\int_0^1 d\sigma^1 \hat{e}$ [4,5,7]. With this choice, the area of the fiducial world-sheet, $\int d^2\sigma \sqrt{\hat{g}}$, equals l.

Begin with the integration over the bosonic embedding coordinates, X^μ. Integration over p+1 constant modes in the Neumann directions, and upon normalizing the measure for infinitesimal variations as in [4] we obtain the result:

$$\int [dX] e^{-S_{SP}[X,\psi,g,\chi]} = 2iV_{p+1}(4\pi^2\alpha')^{-(p+1)/2} l^{(p+1)/2} e^{-R^2 l/4\pi\alpha'} (\det{}'\Delta)_g^{-5} e^{-S_{\rm eff.}[\psi,\chi,\hat{g}]} \quad , \tag{14}$$

where the determinant for the scalar Laplacian is computed with the NN boundary condition on p+1, and DD boundary condition on 9$-p$, coordinates. The overall factor of two accounts for the two possible orientations of the open string. The effective action for the fermionic fields takes the form:

$$S_{\text{eff.}} = \frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{g}[\tfrac{1}{2}\alpha'\bar{\psi}^\mu\gamma^m\partial_m\psi_\mu + \tfrac{\alpha'}{2}(\bar{\chi}_a\gamma^b\gamma^a\psi^\mu)(\bar{\chi}_b\psi_\mu)]$$
$$-\int d^2\sigma\sqrt{g}(\bar{\chi}_a\gamma^m\gamma^a\psi^\mu)(\sigma)\int d^2\sigma'\sqrt{g}(\bar{\chi}_a\gamma_m\gamma^a\psi_\mu)(\sigma')\partial^2_{\sigma\sigma'}G(\sigma,\sigma') \quad . \tag{15}$$

$G(\sigma,\sigma')$ is the Greens function of the scalar Laplacian.

Consider now the gauge fixing of the local symmetries of the effective action. A bosonic deformation of the metric is decomposed into a Weyl transformation, a diffeomorphism continuously connected to the identity, and a change in the length of the boundary of the cylinder. We will gauge both diffeomorphisms and Weyl transformations of the metric. Defining implicitly the measure for infinitesimal variations in the tangent space to the space of metrics as in [4] gives the result:

$$\frac{1}{\text{Order}(\tilde{D})}\int\frac{[dg](\det{}'\Delta)_g^{-5}}{\text{Vol}(\text{Diff}_0\times\text{Weyl})} = \frac{1}{2}\int\frac{[d\delta V]}{\text{Vol}(\text{Diff})_0}\int\frac{[d\delta\phi]}{\text{Vol}(\text{Weyl})}\int_0^\infty dl\ J_b(l;\hat{g})e^{-\frac{10-d}{48\pi}S_L[\phi,g]}(\det{}'\Delta)_{\hat{g}}^{-5} \quad , \tag{16}$$

where J_b is the Jacobian from the change of variables computed in [4]:

$$J_b = \frac{(l/2\pi)^{1/2}\cdot(\frac{2}{l^2})^{1/2}\cdot(\frac{1}{2}l^2\eta^4(\frac{il}{2}))^{1/2}}{(l^3/2\pi)^{1/2}} \quad , \tag{17}$$

in the cylinder metric specified above. We have divided by the order of the subgroup of the disconnected component of the diffeomorphism group, $\tilde{D}$: discrete diffeomorphisms of the world-sheet left invariant under the choice of superconformal gauge [4]. This gives a factor of two in the denominator of Eq. (16), correcting for the two-fold invariance of the measure under the diffeomorphism: $\sigma^1\to-\sigma^1$. The Weyl anomaly of the measure exponentiates to a term proportional to the Liouville action, whose coefficient vanishes in the critical spacetime dimension, d. Consider the result of performing a local supersymmetry and super-Weyl transformation on this expression. This will induce a super-Weyl anomaly. We must simultaneously include the contributions from world-sheet fermions to the Weyl anomaly.

An arbitrary fermionic deformation of the world-sheet gravitino can be decomposed:

$$\delta\chi_m = -D_m\delta\xi + (\delta\zeta)\gamma_m + (\partial_\alpha\chi_m)(\delta\nu^\alpha) \quad , \tag{18}$$

where $\delta\xi$ is an infinitesimal supersymmetry transformation, $\delta\zeta$ is a rescaling of the gravitino, and $\delta\nu$ is a change in a possible supermodulus— a constant two component spinor, ν, on the cylinder. We work in superconformal gauge, invoking super-Weyl transformations in setting $\gamma^m\chi_m$=0 [9]. We make an orthogonal decomposition into infinitesimal deformations parallel (perpendicular) to the gauge slice, respectively, preserving (violating) the restriction to gamma traceless χ_m. In addition, we must separate the contribution from superconformal Killing spinors, $\delta\xi_0$, which leave the world-sheet gravitino unchanged:

$$-D_m(\delta\xi_0) + (\delta\zeta_0)\gamma_m = (-D_m + \tfrac{1}{2}\gamma_m\gamma^n D_n)(\delta\xi_0) = 0 \quad , \tag{19}$$

since the supermodulus, $\delta\zeta_0$=$\frac{1}{2}\gamma^m D_m(\delta\xi_0)$, is in the kernel of the operator D_m. The superconformal Killing spinor, $\delta\xi_0$, is in the kernel of the operator, $P_{1/2}$=$-2D_m$+$\gamma_m\gamma^n D_n$. In the fiducial cylinder metric, D_m=∂_m, and the Killing spinor is simply a constant spinor. Likewise, the supermodulus is also a constant spinor. Constant spinors can only exist on cylindrical world-sheets with both periodic spin structure *and* periodic boundary condition at the endpoints of the open string. Thus, for α=β=1, we have one superconformal constant spinor and one supermodulus to be accounted for in the measure. Performing the change of variables in Eq. (18) we can write:

$$\int_{[\beta,\alpha]}\frac{[d\delta\chi_m]}{\text{Vol(sWeyl}\times\text{sDiff)}} = \int\frac{[d\delta\xi]'}{\text{Vol(sDiff)}_0}\int\frac{[d\delta\zeta]}{\text{Vol(sWeyl)}}\int_{[\beta,\alpha]}[d\delta\nu]J_f^{(\beta,\alpha)} \quad , \tag{20}$$

where the integration over a supermodulus is absent unless α=β=1. A norm in the tangent space to the space of the spin 3/2 field χ_m invariant under both reparameterizations and local Lorentz transformations can be written as follows:

$$|\delta\chi_m|^2 = -i\int d^2\sigma\sqrt{g}(\delta\bar{\chi}^m)(\delta\chi_m) = 2\int d^2\sigma\sqrt{g}(\delta\chi_1^+\delta\chi_1^- + \delta\chi_2^+\delta\chi_2^-) \quad , \tag{21}$$

where the measure in the path integral is normalized as in [4]:

$$1=\int[d\delta\chi_m]e^{-|\delta\chi_m|^2/2}\equiv\int[(d\delta\chi_1^+)(d\delta\chi_1^-)(d\delta\chi_2^+)(d\delta\chi_2^-)]e^{-|\delta\chi_m|^2/2} \quad . \tag{22}$$

We can likewise define a norm in the tangent space to the space of ξ:

$$|\delta\xi|^2=-i\int d^2\sigma\sqrt{g}(\delta\bar{\xi})(\delta\xi), \qquad 1\equiv\int[(d\delta\xi^+)(d\delta\xi^-)]e^{-\int d^2\sigma\sqrt{g}(\delta\xi^+)(\delta\xi^-)} \quad . \tag{23}$$

Separating the ordinary Grassmann integral over the superconformal Killing spinor, ξ_0, gives:

$$1=\int[d\delta\xi_0]e^{-\frac{i}{2}(\delta\bar{\xi}_0)(\int d^2\sigma\sqrt{g})(\delta\xi_0)}\int[(d\delta\xi^+)'(d\delta\xi^-)']e^{-|\delta\xi|^2/2}=l\int[(d\delta\xi^+)'(d\delta\xi^-)']e^{-|\delta\xi|^2/2} \quad . \tag{24}$$

Likewise, the ordinary Grassmann integration over a supermodulus gives:

$$\int[d\delta\nu]e^{-\frac{i}{2}(\delta\bar{\nu})(\int d^2\sigma\sqrt{g})(\delta\nu)}=l \quad . \tag{25}$$

This last term is only necessary when computing the path integral with α=β=1. The measure for the path integral with constant spinors on the worldsheet is discussed in appendix B. For a configuration of parallel and static Dbranes, the presence of the constant mode on the world-sheet gives a vanishing result for the path integral. Henceforth, we restrict our discussion to the cases $(\beta,\alpha)\neq(1,1)$, leaving a discussion of the α=β=1 results to appendix B. Note that the norms in Eqs. (21)–(24), are neither Weyl, nor super-Weyl, invariant. Weyl and super-Weyl transformations generate variations orthogonal to the gauge slice defined by gamma traceless χ. Consequently, both symmetries are anomalous, giving contributions to the path integral measure which will exponentiate to terms proportional to the supersymmetric Liouville action [10].

The result of a local supersymmetry transformation plus super-Weyl scaling on the expression in Eq. (16) is a variation outside the superconformal gauge slice: S_L supersymmetrizes to the super-Liouville action and the super-Weyl anomaly exponentiates to a local renormalizable term proportional to the induced $\gamma^m(\delta\chi_m)$. The integration over diffeomorphisms continuously connected to the identity gives the volume of the Weyl group, cancelled by the term in the denominator. Thus, under a local supersymmetry and super-Weyl transformation, the variation of Eq. (16) gives:

$$\int\frac{[d\delta\phi]}{\mathrm{Vol(Weyl)}}\int_0^\infty dl\;J_b(l;\hat{g})e^{-\frac{10-d}{48\pi}S_{SL}[\phi,\zeta,g]}(\det{}'\Delta)_{\hat{g}}^{-5} \quad , \tag{26}$$

where $S_{SL}[\phi,\zeta,g]$ is the super-Liouville action. In the fiducial cylinder metric,

$$S_{SL}[\phi,\zeta,g]=\int d^2\sigma\sqrt{g}[\tfrac{1}{2}g^{mn}\partial_m\phi\partial_n\phi+\bar{\zeta}\gamma^m\partial_m\zeta+\mu_1e^{\phi}+\mu_2\bar{\zeta}\zeta e^{\phi/2}] \quad , \tag{27}$$

where μ_1 and μ_2 are induced violations of, respectively, the Weyl and super-Weyl invariance of the measure.

Likewise, consider the measure for the fermionic fields in the path integral. In the absence of a supermodulus, the effective action for the fermi fields given in Eq. (15) reduces to the free field action for the matter fermions alone: the world-sheet gravitino can be entirely gauged away. As with the gauge fermions, there are no constant modes for the matter fermion on a world-sheet of cylindrical topology when $\alpha,\beta\neq1$. Integrating over the ψ^μ with norm,

$$|\delta\psi^\mu|^2=-i\int d^2\sigma\sqrt{g}(\delta\bar{\psi}^\mu)(\delta\psi_\mu), \qquad 1=\int[(d\delta\psi^{\mu+})(d\delta\psi^{\mu-})]e^{-\int d^2\sigma\sqrt{g}(\delta\psi^{\mu+})(\delta\psi^-_\mu)} \quad , \tag{28}$$

gives $(\det\gamma^m\partial_m)_g^d$, in the fiducial cylinder metric. Thus, for $(\beta,\alpha)\neq(1,1)$, we obtain:

$$\int\frac{[d\chi_m](\det\gamma^m\partial_m)_g^{10}}{\mathrm{Vol(sDiff\times sWeyl)}}=\int\frac{[d\delta\xi]}{\mathrm{Vol(sDiff)}}\int\frac{[d\delta\zeta]}{\mathrm{Vol(sWeyl)}}J_f(l;\hat{g})[\det(\gamma^m\partial_m)]_{\hat{g}}^{10}e^{-\frac{10-d}{96\pi}S_{SL}[\phi,\zeta,g]} \quad . \tag{29}$$

Combining the expressions in Eqs. (27) and (29) we see that, in the critical spacetime dimension, the Liouville, and super-Liouville, modes entirely decouple. The Jacobian J_f describes the change of variables from $\delta\chi_m$ to $(\delta\xi,\delta\zeta)$ and was first computed in [6]. In the absence of a supermodulus and superconformal Killing spinor on the world-sheet, the fermionic Jacobian is simply:

$$J_f=(\det P_{1/2}^\dagger P_{1/2})^{-1/2} \quad , \tag{30}$$

where $P_{1/2}^{\dagger}=\partial_m$ on the cylinder, is the adjoint of the operator $P_{1/2}$. The extension for $\alpha=\beta=1$ is discussed in appendix B. Combining with results from Eqs. (14), (17), we obtain the following expression for the amplitude:

$$\mathcal{A}=\frac{i}{2}V_{p+1}(4\pi^2\alpha')^{-(p+1)/2}\int_0^\infty \frac{dl}{l}l^{-(p+1)/2}e^{-R^2l/4\pi\alpha'}\eta(\frac{il}{2})^{-8}\sum_{(\beta,\alpha)\neq(1,1)}C_\alpha^\beta[\det(\gamma^m\partial_m)]^8_{(\beta,\alpha)} \quad , \tag{31}$$

where the contribution from two left-moving and two right-moving component fermions— one each of which is a timelike fermion [2], has been cancelled against the fermionic Jacobian, J_f. We are assuming *identical* boundary condition, β, for all NN, and DD, fermions on a world-sheet with fixed spin structure α.

The fermionic determinants in Eq. (31) can be computed using the method of zeta function regularization. The component world-sheet fermions are complexified into Weyl fermions, equivalently, using bosonization, into chiral bosons [2]. An advantage is that the result can be readily generalized to the modified boundary conditions describing a pair of Dpbranes rotated relative to each other. Upon analytic continuation of a Euclidean embedding coordinate to Minkowskian time, this is equivalent to imposing boundary conditions describing parallel Dpbranes in relative motion. We begin by grouping the eight left-moving component fermions, ψ^{+i}, i=1, $\cdots$, 8, into four left-moving Weyl fermions:

$$\psi^{+1}+i\psi^{+2},\ \psi^{+3}+i\psi^{+4},\ \psi^{+5}+i\psi^{+6},\ \psi^{+7}+i\psi^{+8} \quad . \tag{32}$$

Likewise, complexifying eight right-moving component fermions, ψ^{-i}, gives four right-moving Weyl fermions. This is the world-sheet fermion content of the fermionic string with both left and right moving N=1 world sheet supersymmetries. The open string boundary condition in Eq. (12) reduces the number of independent fermionic degrees of freedom by half, since it relates corresponding left and right moving Weyl fermions at the end-points. Thus, while the 4+4 Weyl fermions are not all independent world-sheet fields, this is a convenient basis in which to express the result. We will work with the type I theory in its T-dual formulation with a type IA, or type IB, supersymmetry, as determined by Dpbranes with even, or odd, $p\leq 9$. For convenience, we will keep the full $SO(8)$ global symmetry of the transverse coordinates, imposing identical boundary conditions and spin structure on all four independent Weyl fermions on the world-sheet. The required chiral determinants can all be obtained from the single functional determinant:

$$\det{}'\Delta^{(\beta,\alpha)}=\prod_{n_1,n_2}{}' (\frac{4\pi^2}{l^2})\left[(n_1+(\alpha-1)/2)^2+\frac{l^2}{4}(n_2+(\beta-1)/2)^2)\right] \quad . \tag{33}$$

computed by the method given in [4,5,7]. The chiral determinants are formally defined by taking a square root. The result is therefore ambiguous up to a phase. We retain the phase ambiguity, absorbing it in the unknown C_α^β. For anti-periodic (periodic) Weyl fermions with β=0(1), and world-sheets with spin structure, α, we obtain the result [4,2,7]:

$$\det{}'\Delta^{(\beta,\alpha)}=\left[q^{-\frac{1}{24}+\frac{\beta^2}{8}}\prod_{m=1}^{\infty}(1+e^{\pi i\alpha}q^{m-\frac{1}{2}(1-\beta)})(1+e^{-\pi i\alpha}q^{m-\frac{1}{2}(1+\beta)})\right]\equiv\frac{\Theta_{(\beta,\alpha)}(0,\frac{il}{2})}{\eta(\frac{il}{2})} \quad . \tag{34}$$

As is shown in appendix B, in the case $\alpha=\beta=1$, the left hand side of Eq. (34) is corrected by a contribution from the measure which vanishes for boundaries on static Dbranes. Inserting the expressions for $(\beta,\alpha)\neq(1,1)$ in Eq. (31) gives [6,2]:

$$\mathcal{A}=\frac{i}{2}V_{p+1}(8\pi^2\alpha')^{-(p+1)/2}\int_0^\infty\frac{dl}{l}e^{-R^2l/2\pi\alpha'}l^{-(p+1)/2}\eta(il)^{-12}\left[C_0^0\Theta^4_{(0,0)}(0,il)+C_1^0\Theta^4_{(0,1)}(0,il)+C_0^1\Theta^4_{(1,0)}(0,il)\right] \quad , \tag{35}$$

where the C_α^β are undetermined phases. Note that we have rescaled by a factor $l\rightarrow 2l$ in writing Eq. (35).

B. Infrared Consistency of the Supersymmetric Annulus

We now come to the interesting issue of determining the phase of the path integral. The discussion that follows is based on ideas taken from [14] and also [15,1,2]. We will show that the following infrared consistency conditions:

- the elimination of the tachyon

- the absence of a static force between the Dbranes

determine two of the three unknown phases in the expression for the supersymmetric annulus given in Eq. (35). It should be emphasized at the outset that these requirements are *insufficient* to ensure infrared finiteness of the perturbative string theory [14,2]. The reason is that the oriented open and closed supersymmetric string has tadpoles for both the dilaton and the Ramond-Ramond scalar fields which are cancelled by contributions to the vacuum amplitude from non-orientable world-sheets in the full unoriented type I$'$ string theory [23,12,2]. More generally, the presence of classical sources in the generic String/M theory background akin to the orientifold planes of the unoriented string may provide a means to cancel the troublesome tadpoles so we will leave this option open. Tadpole cancellation is an essential requirement for an infrared finite theory [14,1]. This is sometimes phrased as the requirement of BPS charge conservation in a compact space. We note that we are taking the point of view that an acceptable theory should have a sensible definition both in a noncompact, and a compactified, spacetime. In the compact space, the flux lines of the associated background field are required to close on a configuration of classical Ramond-Ramond sources. While we believe it likely that the infrared consistency conditions on the supersymmetric annulus amplitude listed above are necessary conditions which must be met by the generic stable background of String/M theory— irrespective of whether this is a background described by orientable or non-orientable world-sheets, we should note here the recent work on unstable brane configurations [24], and on the stabilization of the tachyon in open string field theory [17]. Note also that we have distinguished the BPS condition—the absence of a static force between BPS sources in this example, from the specification of the spacetime supersymmetry. Each of these criteria impacts distinct renormalizability properties of the string theory [2].

The third phase in the annulus amplitude, which we choose to be C_0^0, will be determined by computing the static limit of the long distance potential between Dbranes. Define the Minkowskian potential:

$$\mathcal{A}(r,u) = -i \lim_{T\to\infty} \int_{-T}^{+T} d\tau V[r(\tau),u] \quad , \tag{36}$$

where $r^2{=}R^2{+}u^2\tau^2$, and take the static limit $u{=}0$ [19,20,2,7]. This is a special case of the calculation that follows in section IIID. Due to the no-force condition for the BPS configuration of static and parallel Dbranes the amplitude vanishes. However, if we isolate the contribution from the $(\beta,\alpha){=}(0,0)$ sector alone, we can extract the static long-range Newtonian gravitational potential between the Dbranes. This is required to be attractive, which determines the phase C_0^0. Thus, simple, and universal, infrared consistency conditions determine all of the unknown phases in the supersymmetric annulus.

Let us carry out this procedure explicitly. The long distance physics of the vacuum amplitude is dominated by the lowest lying closed string modes. This can be exposed more clearly by taking the $l{\to}0$ limit of the expression in Eq. (35). Using standard identities for the Jacobi theta functions [22,2] gives:

$$\begin{aligned}\mathcal{A}(r) &= \frac{i}{2}V_{p+1}\int_0^\infty \frac{dl}{l}(8\pi^2\alpha')^{-(p+1)/2}e^{-r^2l/2\pi\alpha'}l^{(7-p)/2}\eta(\frac{i}{l})^{-12}\sum_{(\beta,\alpha)\neq(1,1)} C_\alpha^\beta\ \Theta^4_{(\alpha,\beta)}(0,\frac{i}{l}) \\ &= \frac{i}{2}V_{p+1}(8\pi^2\alpha')^{-(p+1)/2}\int_0^\infty dl e^{-r^2l/2\pi\alpha'}l^{(5-p)/2}q^{-1/2} \\ &\quad\times\left\{q^0(C_0^0+C_0^1)+q^{1/2}[8(C_0^0-C_0^1)+16C_1^0]+O(q)\right\} \quad ,\end{aligned} \tag{37}$$

where $q{=}e^{-2\pi/l}$. The absence of the closed string tachyon appearing at $O(q^{-1/2})$ requires $C_0^1{=}{-}C_0^0$. The absence of a static force between the Dbranes requires that we set $C_1^0{=}{-}C_0^0$. The 8+8 massless states in the $(0,0)$, $(0,1)$ sector contribute to the vacuum amplitude with the opposite spacetime statistics of the 16 massless states in the $(1,0)$ sector. Thus, we discover an underlying spacetime supersymmetry in the vacuum amplitude although we have not required it. While the absence of the static force and the requirement of spacetime supersymmetry are equivalent conditions in this example— the T-dualized type I string in a background of parallel and static Dpbranes, the distinction may be of consequence for non-BPS brane configurations.

It remains to determine the overall phase of the annulus. The massless states in the Neveu-Schwarz sector with spin structure, $(\beta,\alpha){=}(0,0)$, contribute a static long range Newtonian interaction between the Dbranes, analogous to that in the bosonic string [19,20,7]. It should be emphasized that this term is universally present in the one-loop vacuum amplitude of *any* fermionic string theory irrespective of background, prior to possible cancellation by additional contributions to the amplitude. The sign of the potential is determined by the phase C_0^0. Defining the Minkowskian potential as in Eq. (36) and substituting for the phases in Eq. (37) gives:

$$V^{(0,0)}_{\rm static}(r) = -C^0_0\ 2V_p(8\pi^2\alpha')^{-(p+1)/2}\tfrac{1}{2}\int_0^\infty dl e^{-r^2 l/2\pi\alpha'} l^{(5-p)/2}[8+O(q^{1/2})]$$
$$= -8C^0_0\ V_p(8\pi^2\alpha')^{-(p+1)/2}\frac{1}{r^{7-p}}\Gamma(\frac{7-p}{2})(2\pi\alpha')^{(7-p)/2}+\cdots \quad . \tag{38}$$

The factor of eight counts the transverse polarizations of the $\mathbf{8_v}$ multiplet under the $SO(9,1)$ Lorentz group. The potential has a static remnant upon setting u=0. The Newtonian potential is required to be *attractive*, and we can therefore set C^0_0=1. Thus,

$$V^{(0,0)}_{\rm static}(R) = -(d-2)\cdot\frac{1}{R^{7-p}}V_p 2^{2-2p}\pi^{(5-3p)/2}\alpha'^{3-p}\Gamma(\frac{7-p}{2}) \quad , \tag{39}$$

where R is the static separation of the Dpbranes. We emphasize once again that the static interaction in Eq. (39) will be *cancelled* by contributions to the vacuum amplitude from states in the Ramond sector and, in the full string theory, from the unoriented world-sheets. Nevertheless, it has a simple physical interpretation which allows us to use it to determine an unknown phase in the string path integral.

We have shown that infrared consistency conditions determine all three phases in Eq. (35), C^0_0=1, C^0_1=C^1_0=-1, giving an unambiguous expression for the supersymmetric annulus. We emphasize that the ambiguity in phase has been determined by requiring consistency with known *qualitative* features of the long distance physics. On the other hand, the normalization of the string path integral is unambiguously determined [4] leading to the prediction of the numerical coefficient in Eq. (39). We will use these expressions in section IV to make predictions about the short distance physics.

C. Pair Correlation Function of Wilson Loops

In [7], we gave a path integral prescription for the pair correlation function of macroscopic loop observables, $M(C_i)$, $M(C_f)$ in the weakly coupled bosonic string theory, following the earlier work of Cohen et al [5]. The loops, C_i, C_f, are taken to lie in a flat D-manifold, $\mathcal{D}$, and are identified with the closed world-lines of heavy point sources in the gauge theory. The result for the short distance potential is independent of whether $\mathcal{D}$ is the worldvolume of some higher dimensional Dpbrane, the intersection of Dpbranes, or the bulk transverse space orthogonal to some configuration of Dbranes. The key issue is the implementation of boundary reparameterization invariance in the covariant one-loop string path integral. Our interest is in the large loop length limit where the dynamics should be universal, independent of the detailed geometrical parameters of the loops. In [7], we point out that the large loop length dynamics for generic loops is captured rather simply by summing over reparameterizations of loops with one or more marked points. For such maps the sum over reparameterizations of the boundary, ∂M, can be easily implemented in closed form even *prior* to taking the large loop length limit [7]. This gives a well-defined framework for computing the boundary reparameterization invariant pair correlation function which also preserves its *normalization* [4]: this is the key ingredient that enables a numerical prediction for the short distance potential between heavy sources in the gauge theory arising in fluctuations of the vacuum energy density.

Following Cohen et al [5], the tree correlation function for a pair of macroscopic string loops is represented as a path integral over embeddings and metrics on world-sheets of cylindrical topology terminating on fixed curves, C_i, C_f, within the spacetime $\mathcal{D}$, weighted by the locally supersymmetric action given in Eq. (4):

$$< M(C_i)M(C_f) > \ = \ \tfrac{1}{2}\sum_{(\beta,\alpha)\neq(1,1)} C^\beta_\alpha \int_{[C_i,C_f][\beta,\alpha]}\frac{[de][d\chi]}{{\rm Vol}(gauge)_{\partial M}}\int\frac{[dX][d\psi][dg][d\chi]}{{\rm Vol}(gauge)_M}e^{-S_{SP}[X,\psi,g,\chi]-\mu_0\int_{\mathcal{M}}d^2\sigma\sqrt{g}-S_{\rm ren.}} \quad . \tag{40}$$

As has been emphasized in section IIIA-B, we have taken all of the NN, DD, world-sheet fermions to satisfy identical boundary conditions at the end-points σ^2=0, 1, for world-sheets of specified spin structure. We decouple bulk and boundary deformations of the world-sheet fields as in [7], imposing Dirichlet boundary conditions on all of the spatial embedding coordinates, X^μ, μ=1, $\cdots$, 9. Then the boundary conditions on the matter fermions, ψ^μ, are given by Eq. (12). The distinction between the Wilson loop correlation function and the ordinary annulus amplitude computed in section III comes from the inclusion of fluctuations in the world-sheet metric (einbein) on the boundaries of the annulus [3,5,7]. We gauge both boundary diffeomorphisms and local supersymmetry variations on the boundary. Anomalies of the measure under Weyl, and super-Weyl, transformations will, as before, be exponentiated as terms in the effective action proportional to the super-Liouville theory with boundary terms included [10,11]. The quantization of the super-Liouville fields could be performed along the lines of [25], and citations thereof, but we will be interested in the vacuum amplitude in the critical spacetime dimension where the super-Liouville theory decouples.

Consider the measure for einbeins. As was shown in [7], a reparameterization $\delta f(\sigma^1)$, tangential to the boundary induces a non-trivial boundary Jacobian computed in [5,7]. However, a Weyl rescaling of the einbein can always be absorbed in a shift in the Liouville field on the boundary. Consider the variation in the measure for einbeins under a local supersymmetry transformation. From Eqs. (A15)–(A18) of appendix A, we see that the variation in the einbein under a supersymmetry transformation can always be absorbed in a rescaling of the super-Liouville fields, (ϕ, ζ), on the boundary:

$$\delta_S e = 2e^m_a(\delta_S e^a_m) = -2(\delta\bar{\xi})(\gamma^m \chi_m), \qquad \delta_W e = 2\delta\lambda \quad , \tag{41}$$

where δ_S, δ_W, respectively denote the variation under local supersymmetry and Weyl transformations. Likewise, consider the variations in the gravitino on the boundary. In superconformal gauge, setting $\gamma^m\chi_m{=}0$, there are no independent variations of the gravitino on the boundary that have not already been accounted for in the analysis in section IIIB: a variation in χ on the boundary is a departure from superconformal gauge. The resulting super-Weyl anomaly is absorbed in the super-Liouville dynamics. Thus, the sum over boundary deformations of the gravitino in Eq. (40) is pure gauge. We eliminate the contributions to the measure from the zero modes of the Neumann embedding coordinates in the expression for the supersymmetric annulus derived in Eq. (35). The result for the pair correlation function of Wilson loops in the T-dualized type I theory is a remarkably simple extension of the bosonic analysis given in [7]:

$$< M(C_i) M(C_f) >= \int_0^\infty dl e^{-S_{\rm saddle}[\bar{x},\hat{g}]} \, \eta(il)^{-12} \sum_{(\beta,\alpha)\neq(1,1)} C^\beta_\alpha \, \Theta^4_{(\beta,\alpha)}(0, il) \quad . \tag{42}$$

The annulus terminates on fixed curves, C_i, C_f, with fixed spatial separation R in some generic D-manifold, i.e., within the world-volume of a Dpbrane, in the intersection of the world-volumes of two or more Dbranes, or in the bulk space transverse to some configuration of branes. The path integral computes quantum fluctuations about a saddle world-sheet configuration stretched between the loops, C_i, C_f, and the saddle-point action can be computed as in [5,7]. In [7] we focussed on the simplest configurations of coplanar loops which can capture the universal features of the large loop length limit which determines the short distance potential between heavy point sources in the gauge theory. For such a configuration, we showed in [7] that the dominant contribution to the saddle action in the small R, large l limit takes the form, $S_{\rm saddle}{\sim}R^2 l/2\pi\alpha'$, independent of the shape or other geometrical characteristics of the loops.

The reader may wonder if generalizations of this result with new non-trivial fermionic degrees of freedom on the boundary are possible. The answer lies in our understanding of the global structure of supermoduli space. The analysis given above is appropriate for the superconformal gauge fixed perturbative superstring in a Dbrane background. However, since brane dynamics is as yet a poorly understood subject in the wider context of String/M theory, it may be that the global structure of supermoduli space can play an interesting role in the full nonperturbative theory [26].

D. Generic Boundary Conditions on the Annulus

Consider the supersymmetric annulus derived in Eq. (35) with boundaries on a pair of static and parallel Dpbranes. In this section, we sketch the modifications to Eqs. (35) for a configuration of rotated Dpbranes or, by an analytic continuation of a Neumann coordinate, for a pair of Dpbranes in relative motion, previously derived in [19,2]. It is convenient to complexify the coordinates, X^μ, $\mu{=}1, \cdots, 8$, in pairs, decomposing a generic rotation into independent rotations in the four planes, $(1,2)$, $(3,4)$, $(5,6)$, $(7,8)$. The $(1,1)$ fermionic path integral no longer vanishes for generic boundary conditions on the fermions. For the generic rotation in all four planes, all four sectors of the fermionic path integral, $\beta, \alpha{\in}0, 1$, contribute to the one-loop vacuum amplitude [2]. This case is discussed in appendix B. We will consider here the simpler case of rotation in a single plane. The unknown phases in the annulus amplitude are determined by an extension of the infrared consistency conditions described in section IIIB for configurations of moving Dbranes. As a consistency check, we verify that we recover the long distance velocity dependent potential between Dpbranes in relative motion, including the numerical coefficient previously computed in [2]. This result will be adapted in section IV to obtain the supersymmetric pair correlation function of Wilson loops corresponding to closed world-lines of heavy sources in a gauge theory in relative slow motion.

Consider motion in the plane $X^0{=}iX^2$, X^1. The functional determinant for a complex scalar satisfying the V boundary conditions given in Eq. (8) can be obtained using zeta function regularization [2,7]. Likewise for the corresponding Weyl fermion; the fermionic functional determinant with spin structure α and boundary condition β

is simply the expression given in Eq. (34) with nonvanishing argument for the Jacobi theta functions: $\Theta_{(\beta,\alpha)}(\nu,\frac{il}{2})$, with $\nu= ul/2\pi$ [19,2]. Thus,

$$q^{E_0(\beta,u)}\prod_{m=1}^{\infty}(1+z\;q^{m-\frac{1}{2}(1-\beta)})(1+z^{-1}q^{m-\frac{1}{2}(1+\beta)})=\left[\frac{e^{u^2l/2\pi}\Theta_{(\beta,\alpha)}(ul/2\pi,\frac{il}{2})}{\eta(\frac{il}{2})}\right]\quad, \tag{43}$$

where the parameter $z{=}e^{i\pi(\alpha+ul/\pi)}$. The vacuum energy of the Weyl fermion with the V boundary condition is $E_0{=}-\frac{1}{24}+\frac{1}{2}(\frac{iu}{\pi}+\frac{\beta}{2})^2$. Thus, the one-loop vacuum amplitude with boundaries on Dpbranes in relative motion takes the form [19,20,2]:

$$\mathcal{A}(r,u)=\frac{1}{2}V_p\int_0^\infty\frac{dl}{l}(4\pi^2\alpha' l)^{-p/2}\frac{e^{-r^2l/4\pi\alpha'}\eta(\frac{il}{2})^{-9}}{i\Theta_{11}(ul/2\pi,\frac{il}{2})}\sum_{(\beta,\alpha)\neq(1,1)}C^\beta_\alpha\Theta^3_{(\beta,\alpha)}(0,\frac{il}{2})\Theta_{(\beta,\alpha)}(ul/2\pi,\frac{il}{2})\quad. \tag{44}$$

It is convenient to rescale $l{\rightarrow}2l$ in the final result.

The phases in the vacuum amplitude can be determined by infrared consistency conditions. As in section IIIB, we require the absence of a tachyon and the vanishing of the static force between the branes. At long distances, we must recover at order v^4 the attractive $-1/r^{7-p}$ potential between BPS sources required from matching to the low energy type II supergravity theory. The long distance physics of the vacuum amplitude is dominated by the lowest lying closed string modes, exposed by taking the $l{\rightarrow}0$ limit of the expression in Eq. (44). Using standard identities for the Jacobi theta functions [22,2] we can write:

$$\begin{aligned}\mathcal{A}(r,u)&=-\frac{1}{2}V_p\int_0^\infty\frac{dl}{l}(8\pi^2\alpha')^{-p/2}\frac{e^{-r^2l/2\pi\alpha'}l^{(6-p)/2}}{\eta(\frac{i}{l})^9\Theta_{11}(-\frac{iu}{\pi},\frac{i}{l})}\sum_{(\beta,\alpha)\neq(1,1)}C^\beta_\alpha\;\Theta^3_{(\alpha,\beta)}(0,\frac{i}{l})\;\Theta_{(\alpha,\beta)}(-\frac{iu}{\pi},\frac{i}{l})\\&=\frac{1}{2}V_p(8\pi^2\alpha')^{-p/2}\int_0^\infty dl e^{-r^2l/2\pi\alpha'}\frac{l^{(4-p)/2}q^{-1/2}}{2\mathrm{Sin}(-iu)}\\&\quad\times\left\{q^0(C^0_0+C^1_0)+q^{1/2}[(2\mathrm{Cos}(-2iu)+6)(C^0_0-C^1_0)+16C^0_1\;\mathrm{Cos}(-iu)]+O(q)\right\}\quad.\end{aligned} \tag{45}$$

The absence of the closed string tachyon appearing at $O(q^{-1/2})$ requires $C^1_0{=}-C^0_0$. With this choice, a small u expansion of the $O(q^0)$ terms gives:

$$C^0_0(16+8u^2+\frac{8}{3}u^4+O(u^6))+C^0_1(16+8u^2+\frac{2}{3}u^4+O(u^6))\quad. \tag{46}$$

The absence of a static force between the Dbranes requires that we set $C^0_1{=}-C^0_0$. As a consequence the leading contribution to the vacuum amplitude is $O(u^4)$— at which order, spacetime supersymmetry is broken. The non-vanishing coefficient implies the existence of a long range velocity dependent potential between the Dbranes [19,20]. The sign of the potential is determined once we fix the phase C^0_0. Defining the Minkowskian potential as in Eq. (36) and substituting for the phases in Eq. (45) gives:

$$\begin{aligned}V_{\mathrm{long}}(r,u)&=-C^0_0\;2V_p(8\pi^2\alpha')^{-(p+1)/2}\frac{1}{2}\int_0^\infty dl e^{-r^2l/2\pi\alpha'}l^{(5-p)/2}\frac{\tanh(u)}{2i\mathrm{Sin}(-iu)}[2u^4+O(u^6)]\\&=-u^4C^0_0\;V_p(8\pi^2\alpha')^{-(p+1)/2}\frac{1}{r^{7-p}}\Gamma(\frac{7-p}{2})(2\pi\alpha')^{(7-p)/2}+O(u^6)\quad.\end{aligned} \tag{47}$$

The potential is required to be *attractive* which implies $C^0_0{=}1$. Thus, we recover the potential between Dpbranes including the numerical coefficient previously computed in [2]:

$$V_{\mathrm{long}}(r,u)=-\frac{u^4}{r^{7-p}}V_p2^{2-2p}\pi^{(5-3p)/2}\alpha'^{3-p}\Gamma(\frac{7-p}{2})+O(u^6)\quad. \tag{48}$$

Thus, all three phases in Eq. (44) are determined, $C^0_0{=}1$, $C^1_0{=}C^0_1{=}-1$, and we have an unambiguous expression for the amplitude.

An extension of these arguments can be applied to more complicated non-BPS brane configurations. Brane configurations which break one-half (one-quarter) of the spacetime supersymmetries can be distinguished by requiring that an order v^2 velocity dependent force is respectively absent (present). Similar arguments apply to configurations of mixed, intersecting, or rotated Dbranes: from the low energy correspondence to supergravity, we infer the qualitative form of the long distance potential. This is then applied as an infrared consistency condition on the unknown phases in the supersymmetric annulus.

IV. SHORT DISTANCE POTENTIAL BETWEEN HEAVY SOURCES

In an earlier work [7], we showed that there exists a short distance interaction between heavy sources in a gauge theory traversing fixed spacetime paths in some generic background of the bosonic string. The potential arises in fluctuations in the vacuum energy density. The same phenomenon can be observed in the supersymmetric T-dualized type I string theory. We will obtain in this section an expression for the short distance potential between heavy sources in a supersymmetric gauge theory. Our results are derived in a systematic small velocity short distance double expansion, following an analogous treatment of the short distance potential in bosonic string theory in [7]. The potential is extracted from the supersymmetric pair correlation function of Wilson loops, in the limit of large loop lengths and small spatial separations.

We begin with the pair correlation function of Wilson loops corresponding to world-lines of heavy sources in relative collinear motion with nonrelativistic velocity v in the direction X^1. For coplanar loops, this mimics straight line trajectories in the Euclideanized $X^0= iX^2$, X^1 plane: $r^2=R^2+v^2\tau^2$, for small separations r. The modifications to the expression given in Eq. (42) for these boundary conditions are straightforward, following the results of section IIID. In the large loop length limit, we have:

$$< M(C_i)M(C_f) >= \int_0^\infty dl \frac{e^{-r^2 l/2\pi\alpha'}\eta(il)^{-9}}{i\Theta_{11}(ul/\pi, il)} \sum_{(\beta,\alpha)\neq(1,1)} C_\alpha^\beta \; \Theta^3_{(\beta,\alpha)}(0,il)\Theta_{(\beta,\alpha)}(ul/\pi,il) \quad . \tag{49}$$

The short distance dynamics is dominated by the lowest lying modes in the open string spectrum. We define the Minkowskian potential [7]:

$$< M(C_i)M(C_f) >= -i \lim_{T\to\infty} \int_{-T}^{+T} d\tau V_{\rm eff.}[r(\tau), u] \quad . \tag{50}$$

The short distance regime is exposed by expanding the integrand in Eq. (49) in powers of $q{=}e^{-2\pi l}$. Substituting for the phases, we can express the Minkowskian potential at short distance in a small q expansion as in [7]:

$$\begin{aligned} V_{\rm eff.}(r,u) &= 2(8\pi^2\alpha')^{-1/2} \int_0^\infty dl \; \tanh(u) l^{1/2} \frac{e^{-r^2 l/2\pi\alpha'}\eta(il)^{-9}}{\Theta_{11}(ul/\pi, il)} \sum_{(\beta,\alpha)\neq(1,1)} C_\alpha^\beta \Theta^3_{(\beta,\alpha)}(0,il)\Theta_{(\beta,\alpha)}(ul/\pi,il) \\ &= -(8\pi^2\alpha')^{-1/2} \int_0^\infty dl \; e^{-r^2 l/2\pi\alpha'} \frac{l^{1/2}\tanh(u)}{q^{1/2}{\rm Sin}(ul)} \\ &\quad \times \Big[(1+2q^{1/2})^3(1+2q^{1/2}{\rm Cos}(2ul)) - (1-2q^{1/2})^3(1-2q^{1/2}{\rm Cos}(2ul)) - 16q^{1/2}{\rm Cos}(ul) + O(q)\Big] \quad . \end{aligned} \tag{51}$$

The leading non-vanishing terms in this expression, due to massless exchange, contribute at order q^0:

$$V(r,u) = -(8\pi^2\alpha')^{-1/2} \int_0^\infty dl \; e^{-r^2 l/2\pi\alpha'} \frac{l^{1/2}\tanh(u)}{{\rm Sin}(ul)} \left[12 + 4{\rm Cos}(2ul) - 16{\rm Cos}(ul)\right] \quad . \tag{52}$$

We have assumed small velocities $v{=}\tanh(u){\simeq}u$. We now perform a resummation of the integrand in the variables r,u, as in [7]. The regime of validity is determined by the behavior of the cosecant function. We perform a Taylor expansion in the first half-period of its argument, $0{\le}ul{<}\pi$. For sufficiently small u values the oscillations in the integrand are increasingly rapid, smearing out the integral [2]. As in the analysis of the vacuum amplitude for the bosonic string in [7], we note that the contribution from the integration domain $ul{\ge}\pi$ can always be bounded, or evaluated by numerical integration, as a self-consistency check on the approximation. This check provides an upper limit, u_+, on the permissible velocities. With this restriction, the contribution from the domain $l{>}\pi/u_+$ can be dropped and we suppress it in what follows. The potential takes the form:

$$V(r,u) = -(8\pi^2\alpha')^{-1/2} \int_0^{\pi/u_+} dl \; e^{-r^2 l/2\pi\alpha'} l^{-1/2}\tanh(u)/u \left[\sum_{k=1}^\infty C_k (ul)^{2k} + \sum_{k=1}^\infty\sum_{m=1}^\infty C_{k,m}(ul)^{2(k+m)}\right] \quad , \tag{53}$$

where the coefficients in the summation are given by:

$$\begin{aligned} C_k &= \frac{4(-1)^k(2^{2k}-4)}{(2k)!} \\ C_{k,m} &= \frac{8(-1)^k(2^{2m-1}-1)}{(2k)!(2m)!} \; |B_{2m}| \; (2^{2k}-4) \quad . \end{aligned} \tag{54}$$

The B_{2m} are the Bernoulli numbers. Note that the k=1 term vanishes in both sums and the leading velocity dependence of the amplitude is $O(u^4)$. Integrating over l gives a systematic expansion for the potential in powers of u^2/r^4. As in [7], we identify a dimensionless scaling variable, $z=r^2_{\rm min.}/r^2$, where $r^2_{\rm min.}=2\pi\alpha' u$. The velocity dependent corrections to the potential between heavy sources in the supersymmetric gauge theory are succinctly expressed as a convergent power series in the *single* dimensionless variable z:

$$V(r,u) = -(8\pi^2\alpha')^{-1/2}\tanh(u)/u\cdot r^{-1}(2\pi\alpha')^{1/2}[\sum_{k=1}^{\infty} C_k z^{2k}\gamma(2k+1/2,\pi/z)$$
$$+\sum_{k=1}^{\infty}\sum_{m=1}^{\infty} C_{k,m} z^{2(k+m)}\gamma(2(k+m)+1/2,\pi/z)] \quad . \tag{55}$$

Note that the potential takes the form of a scale invariant $1/r$ fall-off, contributed by the bosonic degrees of freedom [7], multiplicatively corrected by a convergent power series in z:

$$V(r,u) = -\frac{\tanh(u)/u}{\Gamma(\frac{1}{2})}\frac{1}{r}\left[z^4\gamma(\frac{9}{2},\pi/z)+O(z^6)\right] \quad , \tag{56}$$

indicative of its origin in fluctuations of the vacuum energy density. Recall that the regime of validity for the double expansion in small velocities and short distances is z<1, u<u_+. The scale factor z determines the magnitude of the velocity dependent corrections and, therefore, the accuracy of the expansion. For a given accuracy, with fixed z value, we can probe arbitrarily short distances r by simultaneously adjusting the velocity. The power series corrections in the superstring are qualitatively similar, but much simpler than the analogous series in the bosonic string theory [7]: the analogous result in a nonsupersymmetric gauge theory receives corrections in the variables z^2, uz/π, and u^2. The leading term in the potential between heavy sources in a supersymmetric gauge theory is therefore $O(u^4/r^9)$:

$$V(r,u) = -\frac{u^4}{r^9}2^4\pi^{7/2}\alpha'^4\Gamma(\frac{9}{2})+O(u^6) \quad . \tag{57}$$

V. CONCLUSIONS

We have given a derivation from first principles of both the normalization and the phase of the supersymmetric annulus in the generic flat D-manifold background of the type I and type II string theories. The normalization of the string path integral is determined by its symmetries [4]. As a consequence, one can extract numerical predictions from one-loop string amplitudes, free from any dependence on the string coupling constant. We have shown in this paper that phase ambiguities in the fermionic string path integral can be eliminated by the imposition of simple, and universal, infrared consistency conditions on *qualitative* features of the long distance physics, by matching to an appropriate supergravity theory. This prescription gives the same result as the usual GSO projection in the superstring, but has the hope of wider applicability to generic backgrounds of String/M theory. We note that we have emphasized the BPS conditions over supersymmetry. This is in keeping with the broader goal of understanding the self-Duality of String/M theory in generic backgrounds for the Ramond-Ramond antisymmetric tensor fields associated with Dbrane charges [2,16]. The preliminary results given here need to be explored in a wider context, extended to an understanding of the consistency conditions that suffice to ensure tadpole cancellation [14,15,1] in generic backgrounds of String/M theory.

Extending our earlier results for the bosonic string [7], we have shown that heavy point sources in a supersymmetric gauge theory in slow relative motion have an attractive, and velocity dependent, interaction at short distances. The potential can be expressed as a convergent power series in the single dimensionless variable $z=r^2_{\rm min.}/r^2$, where $r^2_{\rm min.}=2\pi\alpha' u$ is the minimum distance probed in this approximation, valid for small velocities and short distances in the regime $2\pi\alpha' u << r^2 << 2\pi\alpha'$. It would be gratifying if this result could be exploited as a window into the short distance physics of String/M theory.

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APPENDIX A: SPINOR CONVENTIONS AND LOCAL SYMMETRIES

In this appendix we establish our conventions, simultaneously deriving the Brink-Di Vecchia-Howe-Deser-Zumino action for the fermionic string with Minkowskian signature metric [8]. The classical action is invariant under both reparameterizations and N=1 world-sheet supersymmetry transformations. The Euclidean action used in the string path integral is obtained by an analytic continuation. We consider free massless spinor fields in a two dimensional Minkowskian space parameterized (t, x) with metric:

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \qquad \mathcal{J}^{10} = \tfrac{i}{4}[\gamma^0, \gamma^1] = \tfrac{i}{2}\gamma \quad . \tag{A1}$$

$\mathcal{J}^{10}$ is the sole generator of Lorentz transformations in two dimensions and the matrix γ projects onto spinors of definite chirality. We choose a representation with real gamma matrices. Thus,

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A2}$$

Fermion bilinears that transform as scalars under Lorentz transformations are obtained by identifying a matrix β such that:

$$\beta(D(\Lambda))^\dagger\beta = D(\Lambda)^{-1}, \quad \beta(\mathcal{J}^{\mu\nu})^\dagger\beta = \mathcal{J}^{\mu\nu}, \qquad \text{where } D(\Lambda) \equiv e^{\frac{i}{2}(\omega_{\mu\nu}\mathcal{J}^{\mu\nu})} \quad . \tag{A3}$$

Note that with this choice of gamma matrices the Lorentz generator acts non-unitarily in the spinor representation: $(D(\Lambda))^\dagger \neq D(\Lambda)^{-1}$. Thus, β must be chosen to satisfy the conditions:

$$\beta(\gamma^\mu)^\dagger\beta = -\gamma^\mu, \qquad \beta(\mathcal{J}^{\mu\nu})^\dagger\beta = \mathcal{J}^{\mu\nu} \quad , \tag{A4}$$

with solution β=$i\gamma^0$. It is easy to verify that the fermion bilinear:

$$\psi^+\beta\psi \to \psi^+(D(\Lambda))^\dagger\beta D(\Lambda)\psi = \psi^+\beta(D(\Lambda)^{-1}D(\Lambda))\psi = \psi^+\beta\psi \equiv \bar{\psi}\psi \quad , \tag{A5}$$

is Lorentz invariant. Defining components, $(\psi_\mu)^T$= (ψ^+_μ, ψ^-_μ), the free fermion Lagrangian on flat world-sheets can be written in component form:

$$\mathcal{L} = -\bar{\psi}^\mu\gamma^m \pm \psi^-_\mu = i(\psi^{-\mu})^*(\partial_0 + \partial_1)\psi^-_\mu + i(\psi^{+\mu})^*(\partial_0 - \partial_1)\psi^+_\mu \quad . \tag{A6}$$

Notice that, with the conventions above, charge conjugation is defined by the Majorana condition, ζ^*=$\gamma\zeta$, with spinors ζ and $\gamma(\zeta)^*$ transforming identically under Lorentz transformations:

$$\gamma(\gamma^\mu)\gamma^{-1} = \gamma(\gamma^\mu)\gamma = -(\gamma^\mu)^*, \qquad \gamma(\mathcal{J}^{\mu\nu})\gamma^{-1} = -(\mathcal{J}^{\mu\nu})^* \quad . \tag{A7}$$

We can therefore choose the component fermions, ζ^+, ζ^-, to be real.

Analytically continuing to world-sheets with Euclidean signature, we can set σ^1=it, σ^2=x. Thus, we replace, $\partial_0 \to i\partial_1$, $\partial_1 \to \partial_2$, in Eq. (A6) to obtain the Euclidean Lagrangian in component form:

$$\mathcal{L}_\mathcal{E} = \psi^{-\mu}(\partial_1 - i\partial_2)\psi^-_\mu + \psi^{+\mu}(\partial_1 + i\partial_2)\psi^+_\mu \quad . \tag{A8}$$

It is easy to verify that Eq. (A8) can be recovered from the Lagrangian, $\mathcal{L}_\mathcal{E}$=$\bar{\psi}^\mu\gamma^a\partial_a\psi_\mu$, $\bar{\psi}$≡$(\psi)^T\gamma^1$ with the following choice of gamma matrices with Euclidean metric:

$$\gamma^1 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \qquad \gamma^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^5 = \gamma = -i\gamma^1\gamma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A9}$$

From the equation of motion, it is clear that the component fermions, ψ^+, ψ^-, transform, respectively, as left-handed, and right-handed, spinors on the world-sheet. We remark that our spinor conventions correspond to those used in the text [2]. The reader can verify the identities:

$$\bar{\xi}\chi = (\xi)^T\gamma^1\chi = i(\chi^+\xi^- - \xi^-\chi^+) = \bar{\chi}\xi, \quad \bar{\xi}\gamma^a\chi = -\bar{\chi}\gamma^a\xi, \quad \bar{\xi}\gamma^a\gamma^b\chi = \bar{\chi}\gamma^b\gamma^a\xi \quad . \tag{A10}$$

The free fermion Lagrangian can be extended to a two-dimensional action invariant under a local N=1 supersymmetry following [8]. We must be careful to retain any boundary terms resulting from an integration by parts since our interest is in the classical action for world-sheets with boundary. Appending d free fermions to the bosonic string world sheet gives the free field action:

$$S_0 = \frac{1}{4\pi\alpha'}\int d^2\sigma[\partial^a X^\mu \partial_b X_\mu + \alpha'(\psi^{-\mu}(\partial_1 - i\partial_2)\psi^-_\mu + \psi^{+\mu}(\partial_1 + i\partial_2)\psi^+_\mu)] \quad . \tag{A11}$$

A variation of the free field action under the global supersymmetry transformation:

$$\delta X^\mu = \sqrt{\tfrac{\alpha'}{2}}(\bar\xi\psi^\mu), \quad \sqrt{\tfrac{\alpha'}{2}}(\delta\psi^\mu) = \tfrac{1}{2}(\partial_a X^\mu)\gamma^a\xi, \quad \sqrt{\tfrac{\alpha'}{2}}(\delta\bar\psi^\mu) = -\tfrac{1}{2}\bar\xi\gamma^a(\partial_a X^\mu) \quad , \tag{A12}$$

results in the variation,

$$2\pi\alpha'(\delta\mathcal{L}_\prime) = -\sqrt{\tfrac{\alpha'}{2}}\left\{\partial_a[\tfrac{1}{2}(\partial_b X_\mu)(\bar\xi\gamma^a\gamma^b\psi^\mu)] + (\partial_a\bar\xi)(\partial_b X_\mu)(\bar\xi\gamma^a\gamma^b\psi^\mu)\right\} \quad . \tag{A13}$$

Note that the second term vanishes for a covariantly constant spinor supersymmetry parameter, ξ. We can identify the Noether current, $J^a{=}(\partial_b X^\mu)(\gamma^b\gamma^a\psi_\mu)$, and introduce a fermionic source term in the Lagrangian: $\bar\chi_a J^a$, where $\partial_a\xi{\equiv}{-}(\delta\chi)$ [8]. The resulting variations close upon inclusion of a new term quartic in the fermions. The result is the Deser-Zumino-Brink-De Vecchia-Howe action for the fermionic string [8]. Introducing the world-sheet metric, $g_{mn}{=}e^a_m e_{na}$, we can write the action in covariant form:

$$S = \frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{g}[\tfrac{1}{2}g^{mn}\partial_m X^\mu\partial_n X_\mu + \tfrac{1}{2}\alpha'\bar\psi^\mu\gamma^m\partial_m\psi_\mu + \sqrt{\tfrac{\alpha'}{2}}(\bar\chi_a\gamma^m\gamma^a\psi^\mu)(\partial_m X_\mu) + \tfrac{\alpha'}{4}(\bar\chi_a\gamma^b\gamma^a\psi^\mu)(\bar\chi_b\psi_\mu)] \quad , \tag{A14}$$

invariant under the local supersymmetry transformations:

$$\begin{aligned} \delta X^\mu &= \sqrt{\tfrac{\alpha'}{2}}(\delta\bar\xi)\psi^\mu \\ \sqrt{\tfrac{\alpha'}{2}}(\delta\psi^\mu) &= \tfrac{1}{2}(\partial_a X^\mu)\gamma^a\delta\xi + \tfrac{1}{2}\sqrt{\tfrac{\alpha'}{2}}(\psi^\mu\chi_m)\gamma^m\delta\xi \\ \delta\chi_m &= -D_m(\delta\xi) \\ \delta e^a_m &= -(\delta\bar\xi)\gamma^a\chi_m \quad . \end{aligned} \tag{A15}$$

The action is invariant under both reparameterizations and supersymmetry transformations on the world-sheet. The constraints that arise from making these compatible on the boundary will be discussed in Section II. Let us briefly recall the local invariances of the world-sheet action in the bulk. Under a diffeomorphism of the world-sheet coordinates parameterized by δV^m, we have:

$$\begin{aligned} \delta e^a_m &= \delta V^n(\partial_n e^a_m) + e^a_n(\partial_m\delta V^n) \\ \delta\chi_m &= \delta V^n(\partial_n\chi_m) + \chi_n(\partial_m\delta V^n) \\ \delta X^\mu &= \delta V^n(\partial_n X^\mu) \\ \delta\psi^\mu &= \delta V^n(\partial_n\psi^\mu) \quad . \end{aligned} \tag{A16}$$

In the tangent space at any point of the manifold, we can perform independent Lorentz rotations, $\delta\omega$:

$$\begin{aligned} \delta e^a_m &= (\delta\omega)\epsilon^{ab}e^b_m \\ \delta\chi_m &= \tfrac{1}{2}(\delta\omega)\gamma^5\chi_m \\ \delta\psi^\mu &= \tfrac{1}{2}(\delta\omega)\gamma^5\psi^\mu \quad , \end{aligned} \tag{A17}$$

which leave world-sheet scalars invariant. The scalars are also invariant under rescalings of the world-sheet metric and gravitino. A Weyl rescaling of the metric, $\delta\Lambda$, induces the variations:

$$\begin{aligned} \delta e^a_m &= (\delta\Lambda)e^a_m \\ \delta\chi_m &= \tfrac{1}{2}(\delta\Lambda)\chi_m \\ \delta\psi^\mu &= -\tfrac{1}{2}(\delta\Lambda)\psi^\mu \quad . \end{aligned} \tag{A18}$$

The action is also invariant under the fermionic rescaling, $\delta\zeta$, of the world-sheet gravitino, $\delta\chi_m{=}(\delta\zeta)\gamma_m$, known as a super-Weyl transformation, which leaves all other world-sheet fields fixed. Note that the total number of bosonic and fermionic gauge parameters are equal: $(\delta\lambda, \delta V^n, \delta\omega)$ and $(\delta\xi, \delta\zeta)$ contain four parameters each. They can be used to locally fix the world-sheet metric, e^a_m, and gravitino, χ_m, to their values in the superconformal gauge [9].

APPENDIX B: CONSTANT SPINORS AND THE PATH INTEGRAL IN THE (β, α)=$(1, 1)$ SECTOR

For the Dbrane backgrounds studied in this paper, with either parallel and static branes, or a relative rotation in a single plane, the contribution to the vacuum amplitude from the path integral with (β, α)=$(1, 1)$ vanishes. As mentioned in the text, this is always true unless one considers rotations in all four transverse planes: $(1, 2)$, $(3, 4)$, $(5, 6)$, and $(7, 8)$. By an analytic continuation, this implies that the contribution from the $(1, 1)$ sector to the potential between point sources derived in section IV vanishes unless we consider a general motion with velocity components in *at least four* spatial directions. This can be compared with the discussion of a pair of D4branes in relative motion in four transverse Dirichlet directions given in [2]. From the point of view of the path integral, the vanishing contribution is due to a Grassmann integration over a constant mode which is absent in the action. For motions in four transverse planes, the constant mode is absent for four of the Weyl fermions on the world-sheet. We will verify in this appendix that the Grassmann integration over the constant mode of the remaining Weyl fermion, $(0, 9)$, can be saturated by an insertion in the path integral that comes from the supermodulus in the $(1, 1)$ sector.

We complexify the component fermions as in Eq. (32) giving a total of five independent Weyl fermions on the world-sheet upon imposing the open string boundary conditions. Consider separating the constant mode, $\psi_0^{\pm i}$, i=1, $\cdots$, 5, with $\psi^{\pm i}$=$\psi_0^{\pm i}$+$(\psi^{\pm i})'$. We have,

$$\int [d\psi] e^{-S(\psi)} \rightarrow \int \prod d\psi_0 \int [d\psi'] e^{-S(\psi')} \quad . \tag{B1}$$

Since the integrand is independent of the constant mode, the Grassmann integral will vanish. We will now show that the fermionic Jacobian, J_f, defined in Eq. (20) has a nontrivial dependence on the supermodulus in the $(1, 1)$ sector. This term can be exponentiated as a correction to the effective action for the fermions with the consequence that the Grassmann integration over *one* pair of constant fermion modes no longer vanishes. If these are the only fermionic zero-modes present, one obtains a non-zero contribution to the vacuum amplitude from the $(1, 1)$ sector.

As discussed earlier, in the $(1, 1)$ sector the cylinder has both a supermodulus and a superconformal Killing spinor, both of which are simply constant spinors on the world-sheet. We consider variations of the gravitino that preserve the gamma tracelessness condition, parallel to the gauge slice. Denoting the supermodulus as ν, a constant two component spinor, we can write,

$$\chi_1 = \nu, \qquad \chi_2 = i\gamma^5 \nu \quad . \tag{B2}$$

A traceless variation of the gravitino can be decomposed as $\delta\chi_m = -D_m \xi' + \chi_{m,\alpha} d\nu^\alpha$, where α labels the two spin-components of ν. Substituting in Eq. (23) gives:

$$|\delta\chi_m|^2 = -i \int d^2\sigma \sqrt{g} (\delta\bar{\xi}' d\nu^\alpha) \begin{pmatrix} -D_m D^m & D^m \chi_{m,\beta} \\ \bar{\chi}_{m,\alpha} D^m & \bar{\chi}_{m,\alpha} \chi^m_{,\beta} \end{pmatrix} \begin{pmatrix} \delta\xi' \\ d\nu^\beta \end{pmatrix} \quad . \tag{B3}$$

Substituting in Eq. (22),

$$1 = \int [d\delta\chi_m] e^{-|\delta\chi_m|^2/2} = J_f(\hat{g}) \int [d\delta\xi_0] \int [d\delta\xi'] \int d\nu e^{-|\delta\chi_m|^2/2} \quad . \tag{B4}$$

Including the contributions from the Grassmann integrations over the super conformal Killing spinor and the supermodulus given in Eqs. (24), (25), and substituting for the Jacobian matrix in Eq. (B3), gives the result:

$$J_f(\hat{g}) = [\det(P_{1/2}^\dagger P_{1/2})]^{-1/2} (1/l^2 + 1)^{-1} \quad . \tag{B5}$$

Thus, the fermionic path integral takes the form:

$$\begin{aligned} &\int_{[1,1]} \frac{[d\delta\chi_m]}{\mathrm{Vol(sWeyl \times sDiff)}} e^{\frac{1}{4\pi} \int d^2\sigma \sqrt{g} (\bar{\chi}^m \psi^\mu)(\bar{\chi}_m \psi_\mu)} \\ &\qquad = \int_{[1,1]} d\nu [\det{}'(P_{1/2}^\dagger P_{1/2})]^{-1/2} (1/l^2 + 1)^{-1} e^{(1/l^2+1)\frac{1}{2\pi} \int d^2\sigma \sqrt{g} (\nu^+ \nu^- \psi^{+\mu} \psi^-_\mu)} \quad , \end{aligned} \tag{B6}$$

where we use component form for the fermions in the action. Integrating over ν, we are left with the following insertion in the path integral for the matter fermions:

$$[\det{}'(P_{1/2}^\dagger P_{1/2})]^{-1/2} \frac{1}{2\pi} \int d^2\sigma \sqrt{g} (\psi^{+\mu} \psi^-_\mu) \tag{B7}$$

As mentioned in the text, the functional determinant, $[\det'(P^\dagger_{1/2}P_{1/2})]^{-1/2}$, precisely cancels the contribution to the amplitude from the non-constant modes of one pair of component fermions. We choose these to be the $(0,9)$ pair. Complexify as described above. Summing on i=1, $\cdots$, 5, the insertion takes the form,

$$\frac{1}{2\pi}\int d^2\sigma\sqrt{g}(\psi^{+i}\psi^{-i}) = \frac{l}{2\pi}\psi_0^{+5}\psi_0^{-5} + \ldots \quad , \tag{B8}$$

where the $\cdots$ denote the dependence on the remaining constant modes, if present. Thus, precisely one pair of constant modes is saturated by the insertion. If no additional constant modes are present, the resulting path integral gives a non-vanishing result. The fermionic oscillator contributions in the $(1,1)$ sector are computed by zeta function regularization as in Eq. (43). As an example, consider the relative motion of a pair of Dpbranes in the directions (X^1, X^3, X^5, X^7), with v_i=tanh(u_i), i=1, $\cdots$, 4. Then the contribution to the annulus from the $(1,1)$ sector takes the form:

$$\mathcal{A}_{[1,1]}(r,u) = \frac{1}{2}V_p\int_0^\infty \frac{dl}{l}(4\pi^2\alpha' l)^{-p/2}e^{-r^2 l/4\pi\alpha'}\prod_{i=1}^{4}\frac{\Theta_{(1,1)}(u_i l/2\pi, \frac{il}{2})}{i\Theta_{11}(u_i l/2\pi, \frac{il}{2})} \quad , \tag{B9}$$

previously derived in [2].

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